MR2900750 17A32 13P10 68W30
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An algorithm for the classification of 3-dimensional complex Leibniz algebras. (English summary)

Linear Algebra Appl. **436** (2012), *no. 9,* 3747–3756.

The classification of Leibniz algebras in low dimensions is obtained for specific classes of Leibniz algebras (solvable, nilpotent, filiform, etc.). The classical methods to obtain the classifications essentially solve the system of equations given by the bracket laws; that is, for a Leibniz algebra over a field $\mathbb{K}$ with basis $\{a_1, \ldots, a_n\}$, the bracket is completely determined by the scalars $c_{ij}^k \in \mathbb{K}$ such that

$$[a_i, a_j] = \sum_{k=1}^{n} c_{ij}^k a_k.$$

The Leibniz algebra structure is determined by means of the computation of the structure constants c_{ij}^k, which satisfy the following equations:

$$\sum_{l=1}^{n} (c_{jk}^l c_{il}^r - c_{ij}^l c_{lk}^r + c_{ik}^l c_{lj}^r) = 0, \quad 1 \le i, j, k, r \le n. \tag{1}$$

The classification problem is very difficult to handle because the space of the system of equations given by the structure constants and the equations (1) becomes very hard to compute, especially for dimensions $n \ge 3$, since it is necessary to solve a system of n^4 equations in n^3 unknowns, causing frequent errors in the literature.

In the present paper, the authors obtain a complete classification of the 3-dimensional Leibniz algebras over the field $\mathbb{C}$. To do this, they present an algorithm using Gröbner bases that decides in terms of the existence of a non-singular matrix P if two Leibniz algebra structures over a finite-dimensional $\mathbb{C}$-vector space are representatives of the same isomorphism class. *Luisa M. Camacho*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.